APPENDIX A – DETAILED MATHEMATICAL FORMULATION

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SECTION 1 OPTIMIZATION OBJECTIVE

The optimization objective fits the least-cost multi-product co-optimization methodology that maximizes economic efficiency and relieves *network congestion* while respecting physical constraints. The mathematical formulation is minimization of market payments which in equivalent form can be expressed as maximization of the economic value of dispatched load.

1.1. Maximization of *Economic Gain*

1.1.1. Maximum economic efficiency of the system operation can be achieved through the least-cost *security constrained dynamic dispatch* (SCDD) with co-optimization of multiple electricity commodities.

1.1.2. *Energy* and *reserve* costs present integrated *bid* and *offer* price curves. The price curves are stepwise functions of procured services, therefore costs are piecewise linear functions of service quantities. With price curves being stepwise, the objective function of *Economic Gain* can be expressed as the objective function in Section 4.4 of the Price Determination Methodology *Market Manual*.

1.1.3. The maximization of the overall system costs can be expressed as an SCDD problem with the following optimization objective function in generalized form:

$\max\limits_{En,AS}\{$ Co-optimization objective as maximization of the *Economic Gain*:

$-\sum\limits_{t\in T}(\rho^t \cdot \sum\limits_{unit\in G} C_{unit}^{En;t}(En_{unit}^t))$ - *Energy offer* costs

$-\sum\limits_{t\in T}(\rho^t \cdot \sum\limits_{unit\in G}\left(C_{unit}^{RegRaise;t}(Reg_{unit}^{Raise;t}) + C_{unit}^{RegLower;t}(Reg_{unit}^{Lower;t})\right))$ - Regulation Raise and Regulation Lower *offer* costs

$-\sum\limits_{t\in T}(\rho^t \cdot \sum\limits_{unit\in G} C_{unit}^{FFCS;t}(FFCS_{unit}^t))$ - Contingency *offer* costs

$+\sum\limits_{t\in T}(\rho^t \cdot \sum\limits_{unit\in L} C_{load}^{En;t}(En_{load}^t))$ - *Load bid* income

$-\sum\limits_{t\in T}(\rho^t \cdot \sum\limits_{load\in L} C_{load}^{curt})$ - *Load* curtailment costs

$-\sum\limits_{t\in T}(\rho^t \cdot \sum\limits_{constr\in ConstraintSet} C_{constr}^{vio}\left(CV_{constr}^t\right)) \quad \}$ - Constraint violation costs

In this formulation, cost terms are related with *Offers* while *Bids* are treated as income – i.e. *Offer* price terms have negative sign in objective.

1.1.4. *Self-scheduled energy* is incorporated into *bid* and *offer* limits during optimization by respective price/quantity pairs (PQ segments), where the quantity is the self-scheduled value, while the price is reflective of the prioritization price for the particular self-schedule type (i.e. schedule priority).

Prices are negative for *offer* self-schedules (i.e. projected *generation*) and positive for *bid* self-schedules (i.e. forecasted *load* quantities).

1.2. Minimization of Market Payments

1.2.1. The formulation in Section 1.1 is equivalent to the Objective Function defined as minimization of Market Payments, where the same terms are used but with opposite signs. Equivalent minimization form can be expressed as:

$\min_{En,AS}\{$	Least-cost co-optimization objective as minimization of market payments:
$+\sum_{t\in T}(\rho^t \cdot \sum_{unit\in G} C_{unit}^{En;t}(En_{unit}^t))$	- *Energy offer* costs
$+\sum_{t\in T}(\rho^t \cdot \sum_{unit\in G}\left(C_{unit}^{RegRaise;t}(Reg_{unit}^{Raise;t}) + C_{unit}^{RegLower;t}(Reg_{unit}^{Lower;t})\right))$	- Regulation Raise and Regulation Lower *offer* costs
$+\sum_{t\in T}(\rho^t \cdot \sum_{unit\in G} C_{unit}^{FFCS;t}(FFCS_{unit}^t))$	- Contingency *offer* costs
$-\sum_{t\in T}(\rho^t \cdot \sum_{unit\in L} C_{load}^{En;t}(En_{load}^t))$	- *Load bid* income
$+\sum_{t\in T}(\rho^t \cdot \sum_{load\in L} C_{load}^{curt})$	- *Load* curtailment costs
$+\sum_{t\in T}(\rho^t \cdot \sum_{constr\in Constraint\,Set} C_{constr}^{vio}\left(CV_{constr}^t\right)) \quad \}$	- Constraint violation costs

Where:

$\rho^t = \frac{\Delta_t}{60}$ is market time interval length (Δ_t) expressed as fraction of one hour (60 minutes)

SECTION 2 SYSTEM CONSTRAINTS

The main operational system requirements consist of power balance, contingency reserves requirements and regulation capability requirements. In addition, *transmission network* power flow *constraints* (base case and *contingency* cases) are also considered System *Constraints*.

2.1. System Power Balance

2.1.1. The system power balance is a common requirement for all short-term forward markets forcing the system power balance at each trading time interval. The shadow cost of *load* balance presents *system marginal price* for *energy* and also detects unit that is system marginal.

2.1.2. System power balance can be expressed as an equality equation with difference between variable *supply* and variable *demand* (price sensitive or curtailable) on one side and firm (forecasted) *demand* and losses on the other side.

$$\sum_{unit \in G} En_{unit}^{t} - \sum_{load \in L} En_{load}^{t} = En_{req}^{t} + En_{loss}^{t} \; ; t \in T$$

2.1.3. In its formulation, the power balance is extended for slack variables for under-generation and over-generation condition as:

$$\sum_{unit \in G} En_{unit}^{t} - \sum_{load \in L} En_{load}^{t} + Q_{UG} = En_{req}^{t} + En_{loss}^{t} + Q_{OG} \; ; t \in T$$

Where:

Q_{UG} and Q_{OG} are slack variables for under and over *generation*.

2.1.4. As the *generation* and *load* terms are function of *bid*/*offer* quantities, the power balance equation can be written as:

$$\sum_{i} \sum_{j} G_{i,j} + Q_{UG} = \sum_{i} DB_{i} + \sum_{a} P_{Loss,a} + Q_{OG}$$

Where:

$\sum_{a} P_{Loss,a}$ is the sum of all transmission losses in the system and the *generation offer* quantities and *load bid* quantities (G and DB variables) include projected *generation* and forecasted *load* terms, respectively.

This is somewhat a simplified formulation, where the whole system is connected by electrically contiguous AC *network* and there are no export/imports to the system.

2.1.5. The *network energy* losses are linearized using incremental loss factors around the base operating point in respect to *generators* and *loads*:

$$En_{loss}^{t} = En_{loss}^{base;t} + \Delta En_{loss}^{t} ; t \in T$$

Where:

$$\Delta En_{loss}^{t} = \sum_{unit \in G} \alpha_{node}^{t} \cdot (En_{unit}^{t} - En_{unit}^{base;t}) - \sum_{load \in L} \alpha_{node}^{t} \cdot (En_{load}^{t} - En_{load}^{base;t}) \; ; t \in T \, .$$

2.1.6. The *energy* requirement can present the sum of fixed *loads* and *generations*, system *load forecast* or actual *energy* imbalance. The *market network model* provides for a mix of self-scheduled and offered *generation* on *supply* side and a mix of forecasted nodal *load* and *load bids* on *demand* side.

2.1.7. *Load offers* are considered to represent delivered *load*. The market *energy* balance can be expressed in terms of loss penalty factors and uninterruptible market *energy* requirement as:

$$\sum_{unit \in G} En_{unit}^{t} / pf_{unit}^{t} - \sum_{load \in L} En_{load}^{t} / pf_{load}^{t} = En_{req}^{t} + \Delta En_{req}^{t}; \, t \in T$$

Where:

$$\Delta En_{req}^{t} = En_{loss}^{base;t} - \sum_{unit \in G} \alpha_{node}^{t} \cdot En_{unit}^{base;t} + \sum_{load \in L} \alpha_{node}^{t} \cdot En_{load}^{base;t} \; ; t \in T$$

and loss penalty factors are calculated as follows:

$pf_{unit}^{t} = 1/(1-\alpha_{node}^{t})$ and $pf_{load}^{t} = 1/(1+\alpha_{node}^{t})$

Utilizing the notation given in Appendix A.1, namely, $\alpha_{node}^{t} = \frac{\partial P_{loss}^{t}}{\partial P_{node}}$ the loss penalty factor term can be written in the form: $pf_{unit}^{t} = \frac{1}{(1 - \frac{\partial P_{loss}^{t}}{\partial P_{node}})}$

pf_{unit}^{t} is also referred to as the *transmission loss factor* (TLF).

Islanded Operation

2.1.8. In accordance to the centralized concept of the system operation, only a single system wide power balance is considered. However, in case of electric islanding condition, or when parts of electric grid are connected only by *HVDC* links, a separate *load* balance equation will be applied for each energized electrical island.

2.1.9. There will be mapping of *nodes* (*loads* and *generators*) to islands. Based on that mapping, SCDD will formulate *load* balance equation for each island. Accordingly, *shadow price* on the relevant *energy* balance constraint will be calculated for each electrical island. In case of islanding, there is no system level power balance but each region has its own power balance equation.

2.1.10. For each electrical island *i* the following equation will be written for a given time interval *t:*

$$\sum_{unit \in Gi} En_{unit}^{t} - \sum_{load \in Li} En_{load}^{t} + Q^{i}{}_{UG} = En_{req}^{i,t} + En_{loss}^{i,t} + Q^{i}{}_{OG} \; ; t \in T$$

2.1.11. During the islanding condition, congestion in one island does not affect the congestion of other islands. Losses are also calculated per island.

2.1.12. In scenario where *grids* are connected only by *HVDC* link, additional terms presenting DC pole injections for each *HVDC* link *dc* connected to particular grid *i* will show in each grid *i load* balance.

$$\sum_{unit \in Gi} En^{t}_{unit} - \sum_{load \in Li} En^{t}_{load} + \sum_{dc \in HVDCi} En^{t}_{dc} + Q^{i}{}_{UG} = En^{i,t}_{req} + En^{i,t}_{loss} + Q^{i}{}_{OG} \; ; t \in T$$

Term En^{t}_{dc} is positive for *HVDC* imports and negative for *HVDC* exports.

2.2. Regional Reserve Requirements

2.2.1. The *reserve requirements* can be specified for each *reserve region*. *Reserve regions* are the same for all *reserves* and for all time intervals. Nevertheless, separate requirements can be specified for each *reserve region*, each *reserve category* and each scheduling time interval. The overall system is treated as a *reserve region.*

Regulation Raise and Regulation Lower Reserve Requirements

2.2.2. The regulation capability is provided through the regulation capacity market segment. Separate minimal requirements for Regulation Raise capacities:

$$\underline{Reg}^{Raise;t}_{ASreq} \leq \sum_{unit \in AS} Reg^{Raise;t}_{unit} \; ; t \in T$$

and maximal and minimal requirements for Regulation Lower capacities:

$$\underline{Reg}^{Lower;t}_{ASreq} \leq \sum_{unit \in AS} Reg^{Lower;t}_{unit} \leq \overline{Reg}^{Lower;t}_{ASreq} \; ; t \in T$$

2.2.3. Only online generating units can be awarded regulation service to contribute to the regional regulation requirements.

2.2.4. The Regulating reserve requirements equations also include slack variables for insufficient regulating reserve.

Contingency Reserve Requirements

2.2.5. Analogously to Regulating Reserve Raise and Regulating Reserve Lower minimal requirements, regional minimum requirements can be specified for other ancillary services (AS) and for each time interval:

$$\underline{Res}^{t}_{ASreq} \leq \sum_{unit \in AS} Res^{t}_{unit} \; ; t \in T$$

2.2.6. The Contingency Reserve requirements equations also include slack variables for insufficient contingency reserve.

2.3. Reserve provider capacity caps

2.3.1. Reserve Provider capacity caps are group *constraints*, where an aggregated award may be less than or equal to a specified value. Capacity caps are defined per:

a. Ancillary Service provider (Market Participant)
b. Class of Ancillary Service providers

2.3.2. In both cases the equation can be written as:

$$\sum_{unit \in AS\ Group} Res_{unit}^{t} \leq \overline{Res}_{AS;\ AS\ Group}^{t}\ ;\ t \in T\ ,$$

Where AS Group can be each affected AS provider or AS provider class.

2.4. AC Power Flow Model

2.4.1. Accurate power flow results presenting physical system operation are essential for market operation. The power balance equations for some *network node k* having incident *nodes* m can be specified in the following form:

$$P_{node}^{k} = V_{node}^{k} \sum_{m \in I_k} V_{node}^{m} [G_{line}^{km} \cos(\theta_{node}^{k} - \theta_{node}^{m}) + B_{line}^{km} \sin(\theta_{node}^{k} - \theta_{node}^{m})]$$

$$Q_{node}^{k} = V_{node}^{k} \sum_{m \in I_k} V_{node}^{m} [G_{line}^{km} \sin(\theta_{node}^{k} - \theta_{node}^{m}) - B_{line}^{km} \cos(\theta_{node}^{k} - \theta_{node}^{m})]$$

2.4.2. The AC power flow equations completely determine the *network* operating state and their solution $[V_{node}^{k;base};\theta_{node}^{k;base}]$ is calculated for all *network nodes*.

2.4.3. This solution is considered as the base *network* state. All nodal power flow injections, line power flows and *network* losses are calculated for the base *network* state. Additionally *network energy* loss sensitivities and *transmission line* shift factors are calculated to provide a linearized AC model for the *network* base state.

2.4.4. The AC power flow respects unit MW limits, MVAR limits, scheduled voltages for local voltage controlled buses and limits on shunt capacitor banks, load tap changer (LTC) taps and phase-shifter taps.

2.4.5. In cases when nodal *loads* include losses the AC power flow uses *load* distribution slack to allocate *network energy* losses. The adjusted *load* schedules present the delivered nodal *loads* corresponding to the *generation* schedules. If *load* schedules present delivered *load* themselves then *network energy* losses are distributed to the *generation* schedules.

Network Loss Model

2.4.6. Summing up all AC power flow nodal balance equations, including *network energy* losses, the system power balance equation in terms of nodal *generation* and *load* schedules is obtained:

$$\sum_{node \in G} P_{node}^{base;t} - \sum_{node \in L} P_{node}^{base;t} = \sum_{node \in G \cup L} P_{node}^{fix;t} + P_{loss}^{base;t}(P_G^{base;t}, P_L^{base;t})\ ;\ t\ \in T$$

2.4.7. Both *generation* and *load* nodal power injections are expressed as positive values. At the same time, the nodal loss sensitivity factors are calculated as derivatives of *network energy* losses in respect to *generation* nodal power injections. Therefore, the *load* sensitivity loss factors are equal to the negative *generation* nodal loss factors.

2.4.8. The loss sensitivity factors are calculated using a reference bus approach. The resulting linearized model for *network* losses can be specified as follows:

$$P_{loss}^{t}(P_G^t, P_L^t) = P_{loss}^{t}\left(P_G^{base;t}, P_L^{base;t}\right) + \sum_{node \in G} \alpha_{node}^{t} \cdot \left(P_{node}^{t} - P_{node}^{base;t}\right) - \sum_{node \in L} \alpha_{node}^{t} \cdot \left(P_{node}^{t} - P_{node}^{base;t}\right);\ t \in T$$

Line Power Flow Limits

2.4.9. Transmission branches/paths congested due to *energy* schedules are considered for both the base case and *contingency* cases. The branch flow MVA limits are translated into MW limits, making the assumption that MVAR branch flows and voltage magnitudes do not change significantly due to active power rescheduling. The MW line flow limits are calculated as:

$$\overline{P}_{line}^{t} = SQRT(\overline{MVA}_{line}^{t} **2 - Q_{line}^{b;t} **2); \quad t \in T\ .$$

2.4.10. The *transmission line* flows are expressed as linearized functions of the nodal power injections around the base operating state using calculated Shift Factors:

$$P_{line}^{t} = P_{line}^{base;t} + \sum_{node \in N} SF_{line}^{node} \cdot \left(P_{node}^{t} - P_{node}^{base;t}\right);\ line \in N;\ t \in T$$

2.4.11. The branch power flows of critical *transmission lines* are limited in both directions:

$$\underline{P}_{line}^{t} \le P_{line}^{t} \le \overline{P}_{line}^{t}; \quad line \in N; \quad t \in T$$

2.4.12. When solving the base case the limit used is the Normal limit. When a *contingency* case is being solved, the flows are checked against the *Contingency* limit. It is required that the *Contingency* limit be the same or greater than the Normal limit.

2.4.13. The set of critical transmission lines is selected according to the percentage of line MW loading. The lines loaded above the specified threshold are included.

2.4.14. The branch power flows equations also include segmented slack variables for limit violation.

2.5. Constraints on HVDC operation

2.5.1. The *HVDC* operation in the optimization problem is modeled by introduction of the concept of *HVDC* Resource. *HVDC* Resource is a modeling vehicle to represent the flow MW and flow direction on the *HVDC* line, as well as other *HVDC* operational *constraints*, like the minimum time required to change the flow direction. The *HVDC* Resource MW schedule (injection) is also representing *network* injection or withdrawal for AC *network* at the DC terminal. The model is illustrated below.

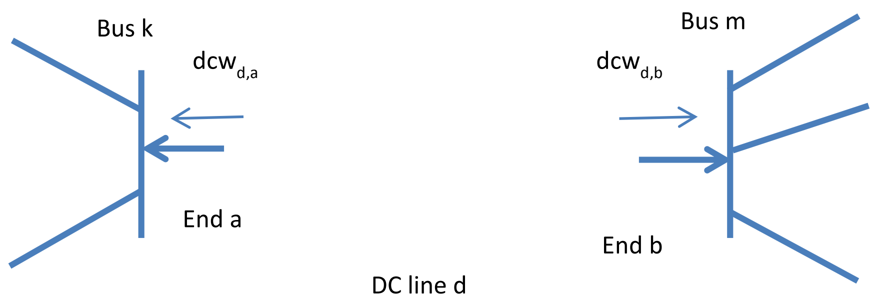

Figure 1: HVDC model

In Figure 1, the DC line *d* goes between the AC buses *k* and *m*. An assignment of ends has been done as End *a* and End *b*. Assume that End *a* is the end where interchange schedules are assumed to flow. This figure shows the model that is implemented in the optimization problem formulation, where the DC line itself has been replaced by a coordinated pair of DC injections.

$dcw_{d,a}^{t}$ This is the MW flow on end *a* of the DC line *d* at time *t*. The sign convention is that flow from the DC line into the AC system is considered negative.

$dcw_{d,b}^{t}$ This is the MW flow on end *b* of the DC line *d* at time *t.* The sign convention is that flow from the DC line into the AC system is considered negative.

2.5.2. In its operation, *HVDC* Resource on both ends of a DC Link can act both as a generator and a *load*, with *HVDC* having three discrete state of operation: no-flow, flow in prevailing direction and flow in direction opposite to the prevailing direction. The MW schedules to the *HVDC* Resources are included in *Load* Balance of each of the electrical islands connected by DC Link. They are also included in *HVDC* flow equation that accounts for losses as described below.

2.5.3. A default DC loss percentage is used to obtain a simplified formulation of DC Link *load* balance.

When the flow is from end *b* to end *a*:

$$\mathrm{dcw_{d,b}^{t}} + \mathrm{dcw_{d,a}^{t}} - \mathrm{dcwlossp_{d,b,a}}/100 \cdot \left(\mathrm{dcw_{d,b}^{t}}\right) = 0$$

When the flow is from end *a* to end *b*:

$$\mathrm{dcw_{d,b}^{t}} + \mathrm{dcw_{d,a}^{t}} - \mathrm{dcwlossp_{d,b,a}}/100 \cdot \left(\mathrm{dcw_{d,a}^{t}}\right) = 0$$

Where:

$dcwlossp_{d,b,a}$ is the default loss percentage.

There are no slack variables associated with above equations. Note that $dcw_{d,a}^{t}$and $dcw_{d,b}^{t}$ variables are unbounded, but subject to DC MW Flow *constraints*. In case of zero losses, above equations essentially state that the DC Link transfer at End *b* is the same as transfer at End *a* by absolute value, but with opposite sign.

If the presence of DC link is the only electrical connection between AC *networks* at End *a* and End *b*, then those *networks* are considered to be separate AC islands, so the network injections at one island do not have shift factors with respect to the AC flow *constraints* in another island.

Directional HVDC Limit

2.5.4. Directional flow *constraints* flow on DC line to be between minimum and maximum flow limit for each direction (if the flow is non zero).

When the flow is from End *a* to End *b*, the constraint is:
$\underline{HVDC_{d,a}^t} \leq dcw_{d,a}^t \leq \overline{HVDC_{d,a}^t}$

When the flow is from End *b* to End *a*, the constraint is:
$\underline{HVDC_{d,b}^t} \leq -dcw_{d,a}^t \leq \overline{HVDC_{d,b}^t}$

Where:

$\overline{HVDC_{d,a}^t}$ and $\underline{HVDC_{d,a}^t}$ are maximum and minimum MW flow limit (positive values) when flow is from *a* to *b*

$\overline{HVDC_{d,b}^t}$ and $\underline{HVDC_{d,b}^t}$ are maximum and minimum MW flow limit (positive values) when flow is from *b* to *a*

2.5.5. Additional binary variables are introduced to enforce directional limits. One binary variable is used to model flow from End *a* to End *b* and another binary variable is introduced to model flow from End *b* to End *a*. When DC transfer variable $dcw_{d,a}^t$ is positive, then the binary variable $dcd_{d,a}^t$ has to be one and the binary variable $dcd_{d,b}^t$ has to be zero. Analogously in case when $dcw_{d,a}^t$ is negative. In case that $dcw_{d,a}^t$ is itself zero, both binary variables have to be zero as well. The upper and lower directional limit on *HVDC* flow can be formulated as:

$$dcw_{d,a}^t \leq \overline{HVDC_{d,a}^t} * dcd_{d,a}^t - \underline{HVDC_{d,b}^t} * dcd_{d,b}^t$$
$$dcw_{d,a}^t \geq \underline{HVDC_{d,a}^t} * dcd_{d,a}^t - \overline{HVDC_{d,b}^t} * dcd_{d,b}^t$$
$$dcd_{d,a}^t + dcd_{d,b}^t \leq 1$$

Where:

$dcd_{d,a}^t$ is a binary variable determining whether the MW flow is from end *a* to end *b* at time interval *t*

$dcd_{d,b}^{t}$ is a binary variable determining whether the MW flow is from end *b* to end *a* at time interval t

2.5.6. The *HVDC* Flow limit equations are soft *constraints* and include slack variables for limit violation, both for minimum and maximum flow limit, and in both directions.

Minimum time needed for HVDC change of flow

2.5.7. Additional constraint applicable to *HVDC* line is the "change of flow direction" constraint. It is described by the minimum time that has to lapse before a power flow on DC line can flow in the opposite direction. Constraint is modeled as form of a minimum down time constraint, i.e. the minimum time *HVDC* Line has to spend in zero flow condition.

2.5.8. The constraint is enforced by the following equations:

$$\sum_{t=t1}^{t1+T_d^{MIN}} dct_{d,a}^{t} \geq T_d^{MIN}$$

$$dct_{d,a}^{t} = 1 - (dcd_{d,a}^{t} + dcd_{d,b}^{t})$$

That are effective for every interval *t1* where the flow changed to zero from being non-zero; i.e. $dct_{d,a}^{t1} - dct_{d,a}^{t1-1} = 1$

Where:

$dct_{d,a}^{t}$ is a helper variable showing that the flow on the DC line is zero at time interval *t*
T_d^{MIN} is the minimum time before the DC line flow can be reversed

2.5.9. In addition to above equations there are boundary conditions considering past as follows:

$dct_{d,a}^{t}$ Variable has counter reflecting the initial condition, i.e. if there is a change of flow to zero from non-zero that occurred in the past, that interval is recorded and the counter is incremented each *real time dispatch* run, while in each subsequent *real time dispatch* run the following is enforced:

$$dct_{d,a}^{t} = 1 \; \forall \; t \leq MAX(0, T_d^{MIN} - t_0)$$

Where:

t_0 is the number of intervals since the flow was last changed from non-zero to zero in the past *real time dispatch* runs.

2.5.10. The change of flow *constraints* are hard *constraints* and cannot be violated in the model. In the case when the *HVDC* line flow direction change is pre-scheduled, the minimum switching time is modified to comply with the schedule (i.e. the line flow direction change schedule is always considered to be feasible).

SECTION 3 BID/OFFER RELATED CONSTRAINTS

The electric *energy* related products are provided from physical resources with limited capacities. In addition to limited amount of available products separately, the capacity limits for resources providing multiple products are included into optimization model. Therefore, the model includes the following limitations for each physical resource and for each time interval:

3.1. Energy Dispatch Limits

3.1.1. Resource *energy* award has to be within the economic limits (*Energy offer/bid* limits).

$$\underline{En}_{res}^{t} \leq En_{res}^{t} \leq \overline{En}_{res}^{t} \quad , \quad res \in G, L$$ Resource *offer/bid* limits

3.2. Regulating Reserve Limits

3.2.1. Regulation Reserve awards (allotments) are less than the upper *offer* limit and are less than the *reserve* ramping capability (the regulation reserve ramping time multiplied by the regulation *ramp rate*).

$$Reg_{unit}^{Raise;t} \leq \min\{\overline{Reg}_{unit}^{Raise;t} ; RR_{unit}^{RegUp} \cdot T_{dom}^{Reg}\}$$

$$Reg_{unit}^{Lower;t} \leq \min\{\overline{Reg}_{unit}^{Lower;t} ; RR_{unit}^{RegDn} \cdot T_{dom}^{Reg}\}$$

3.3. Contingency Reserve Limits

3.3.1. Contingency Reserve awards (allotments) are less than the upper *offer* limit and are less than the contingency reserve ramping capability (contingency reserve ramping time multiplied by the *reserve ramp rate*).

$$Res_{unit}^{t} \leq \min\{\overline{Res}_{unit}^{t} ; RR_{unit}^{Res} \cdot T_{dom}^{Res}\}$$

3.3.2. The *reserve ramp rate* is submitted as part of the *offer*, while *reserve* ramping time is the time required by service definition to reach full response.

3.4. Tie-Break Processing

3.4.1. For scenario of tie-breaking among *offers* for the same service or among *bids* for the same service, the soft 'tie breaking' constraint will be introduced that is enforcing pro-rata equality of awarded block MW quantities. Constraint enforces that the difference between awards for two equally priced blocks, pro-rated by their maximum value, should be equal to zero. As this is equality constraint, two single segment slack variables will be introduced per constraint. For a group of *N* identified blocks that are tied at the same price (from N *offers*, where N is expected to be 2 for most practical cases), and have to be subject to tie break processing, a set of N-1 equation will be written as:

$$\frac{BQ_{k,1}^{i}}{\overline{BQ}_{k,1}^{j}} - \frac{BQ_{k,2}^{j}}{\overline{BQ}_{k,2}^{j}} + \overline{Slack}_{TB,k}^{1} - \underline{Slack}_{TB,k}^{1} = 0$$

3.4.2. Where $BQ_{k,1}^{i}$, $BQ_{k,2}^{j}$ are ith and jth block quantities from first and second *offer* within the group, and $\overline{BQ}_{k,1}^{i}$, $\overline{BQ}_{k,2}^{j}$ are respective block sizes.

$$\frac{BQ_{k,2}^{m}}{\overline{BQ}_{k,2}^{m}} - \frac{BQ_{k,3}^{l}}{\overline{BQ}_{k,3}^{l}} + \overline{Slack}_{TB,k}^{2} - \underline{Slack}_{TB,k}^{2} = 0$$

…

$$\frac{BQ_{k,n-1}^{u}}{\overline{BQ}_{k,n-1}^{u}} - \frac{BQ_{k,n}^{v}}{\overline{BQ}_{k,n}^{v}} + \overline{Slack}_{TB,k}^{n-1} - \underline{Slack}_{TB,k}^{n-1} = 0$$

3.4.3. The slack variables introduced will contribute to the Objective under very low penalty prices (comparing to other penalties), so the constraint can be violated by any other constraint. Analogous equations are written for equally priced *bid* block quantities. These *constraints* can be applied for *energy bid*/*offer* tie-breaking as well as for *reserve bid* tie breaking.

3.4.4. In addition, the tie breaking process will be applied to self-scheduled *generation* (e.g. Tie Breaking of self- scheduled generators in cases of *network* limitation).

3.4.5. To reflect the actual economics of the *market dispatch optimization model*, the “economic” tie breaking will be applied in the model only to resources with the same loss sensitivities (loss penalty factors). Since tie-breaking equations are part of the integral problem formulation, and not post processing, tie breaking solution reflects all the economic characteristics of the model, i.e. congestion costs or AS opportunity costs.

3.4.6. Tie breaking is also applied for *self-scheduled energy* resources in case of curtailment of projected schedule. For this scenario, *constraint violation coefficient* values will be defined for violation of *self-scheduled energy* dispatch in scheduling run. Then in pricing run setup, prices and *self-scheduled energy dispatch schedule* constraint will be set analogously to other soft *constraints*. Pro rata remains the same as for economic *offers*.

3.4.7. There is exception to economic tie breaking rules in curtailment of *self-scheduled energy* dispatch resources, where tie breaking in certain scenarios is performed so that curtailment is performed proportionally to submitted (or forecasted) self-schedules. For additional details please see Appendix A.3.

3.4.8. In case of tie between a *demand bid* and a generator *offer* (with same loss sensitivities), there is no pro-rating, instead the *load* served is maximized by addition of small “incentive term” making the combined *load-generation* award net positive to the objective.

SECTION 4 GENERATING/LOAD RESOURCE CONSTRAINTS

4.1. Energy Capacity Limits

4.1.1. When there is no *reserve offer* from the unit, e*nergy dispatch* has to be within unit operating limits:

$$EnL_{unit}^{t} \leq En_{unit}^{t} \leq EnH_{unit}^{t}$$

4.2. Constant Ramping Limits

4.2.1. The ramping capabilities of *generation* and *load* units are expressed as constant values of maximal Up and Down *Ramp Rates* over the full range of the resource power output. The Up and Down *Ramp Rate* Limits are calculated as a product of maximal Up and Down *Ramp rate* values and the *energy* ramping time domain:

$$RRL_{unit}^{Up} = RR_{unit}^{Up} \cdot T_{dom}^{En}; \quad RRL_{unit}^{Dn} = RR_{unit}^{Dn} \cdot T_{dom}^{En}; \quad unit \in G$$

$$RRL_{load}^{Up} = RR_{load}^{Up} \cdot T_{dom}^{En}; \quad RRL_{load}^{Dn} = RR_{load}^{Dn} \cdot T_{dom}^{En}; \quad load \in L\,.$$

4.2.2. For each *generation* and *load* unit and each time interval the following *energy* Up and Down *Ramp rate* Limits are posted:

$$-RRL_{unit}^{Dn} \leq En_{unit}^{t} - En_{unit}^{t-1} \leq RRL_{unit}^{Up}; \quad unit \in G; t \in T$$

$$-RRL_{load}^{Dn} \leq En_{load}^{t} - En_{load}^{t-1} \leq RRL_{load}^{Up}; \quad load \in L; t \in T\,.$$

4.2.3. The *energy* ramping time domain is dependent on the length of time interval.

4.3. Reserve Model

4.3.1. Core parts of the *Reserve* model are:

a. *Reserve* capacity limits
b. *Reserve* ramping
c. Combined *Energy* and *reserve* capacity limits
d. Combined *Energy* and *reserve* ramping
e. Independent model for Raise and Lower service in each *reserve* category

Resource Reserve capacity limits

4.3.2. In addition to limits imposed by *reserve offer* limits, there are physical unit limits that affect *reserve* award. One example is for fast and slow *reserves* limitation by Governor response. While Governor response also depends on frequency deviation, it is usually one curve provided for Market purpose, where response is given as function of *energy* output only. Typical Governor response curve is provided below:

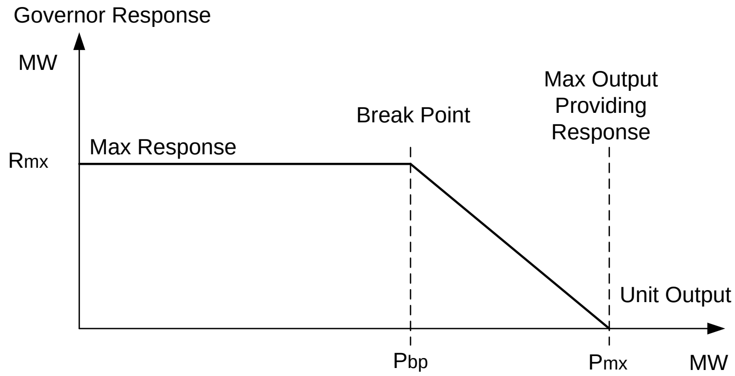

Figure 2: Governor *n* second raise droop characteristic

4.3.3. Each two-piece characteristic comprises:

a. maximum response amount which applies between zero *energy dispatch* and the contracted *energy dispatch* breakpoint and;
b. above the *energy dispatch* breakpoint there is linear decrease in response amount from the contracted maximum response amount down to zero maximum response at the maximum *energy* capacity.

4.3.4. The mathematical formulation using the variable designation from Figure 2 are as follows:
$Res_{unit}^{t} = R_{mx} * (P_{mx} - P^{t})/(P_{mx} - P_{bp}) \ \forall \ P \geq P_{bp}$
$Res_{\mathrm{unit}}^{\mathrm{t}} = \mathrm{P_{mx}} \ \forall \ \mathrm{P} < P_{\mathrm{bp}}$

4.3.5. In addition to maximum quantity, contracted generators might be subject to mandatory governor response, which is modeled as *reserve* self-schedule and protected with penalty in Scheduling Run (i.e. treated as price taker). Such self-schedule also contributes to regional *reserve requirements*.

Resource AS ramping limits

4.3.6. The individual *reserve* ramping constraint can be posted for each resource and each time interval. These *constraints* are expressed in time domain as follows (equation is provided for Regulation Raise, but analogous equation applies for each *reserve*:

$$\frac{Reg_{unit}^{Raise;t}}{RR_{unit}^{RegUp}} \leq T^{AS}; \quad unit \in G; t \in T$$

meaning that the *Reserve* ramping cannot exceed the specified *reserve* ramping (default 5 minutes).

Resource Combined Energy and Reserve Capacity Limits

4.3.7. Multiple market services can be provided by the same resource at the same time, but the total resource capacity is limited. For example, the capacity range of online *generation* resources can be used for *energy*, regulation raise capacity and contingency *reserve*.

4.3.8. The capacity range binding *energy* and *reserve* depends on the services involved. For example, for combined *Energy* and Regulating reserves, the regulating range is binding.

$$En_{unit}^{t} + RegRaise_{unit}^{t} \leq RH_{unit}^{t}$$

$$En_{unit}^{t} - RegLower_{unit}^{t} \geq RL_{unit}^{t}$$

4.3.9. In a scenario when regulating limits are not separately registered for a resource, the operating limits are used instead of regulating in the above equations.

4.3.10. For combined *Energy* and Contingency (Frequency Response) Service, the sum of the scheduled *energy* and the scheduled FCAS response ($PSRaise_{unit}^{t}$) must be less than or equal to the Governor Droop Raise Capacity $\left(GDRH_{unit}^{t}\right)$ of that unit for each of the services:

$$En_{unit}^{t} + PSRaise_{unit}^{t} \leq GDRH_{unit}^{t}$$

4.3.11. Analogous capacity limits are posted on *load* entities. For example of Contingency reserve:

$$En_{load}^{t} - SRes_{load}^{t} \geq EL_{load}^{t}$$

Resource Combined Energy and Reserve Ramping[1]

4.3.12. If the *reserve* awards were dispatched for *contingency,* they would be converted into *energy* that needs to ramp, thus taking away ramping capability of dispatched *energy* award. The *energy* ramping period is the same as *dispatch interval*, so if it was fully utilized for *energy* ramping, there would be no room for additional *energy* ramping needed if *reserve* was activated. Therefore the *reserve* awards have to be taken into account in *energy* ramping model. *Energy* ramping capacity based on *energy ramp rate* is adjusted to address the impact from *reserve* awards. The upward and downward ramping equations can be expressed as:

$$P_i(t) - P_i(t-1) \leq RLU_i^{En}(t) - ASUp_i(t)$$

$$P_i(t-1) - P_i(t) \leq RLD_i^{En}(t) - ASDn_i(t)$$

where: ASUp and ASDn is upward/downward *reserve* impact to *energy* ramping capacity.

[1] Example is provided in Appendix A.3

4.4. Other Operational Modes of Generators, Loads or Similar Facilities

4.4.1. Hybrid resources or other operational modes of generators, loads, or similar facilities include hydro pump storage and non-generating resources (NGR)[2] like batteries, flywheel, compressed air facilities, and other forms of primary energy storage.

4.4.2. Potential models for the treatment of these resources may incorporate variability of both supply offers and demand bids for more optimal economic results.

SECTION 5 MARKET CLEARING PRICES

5.1. The SCDD model calculates constraint shadow costs as a byproduct of the optimization process. Those shadow costs are directly taken from optimization solution constraint dual variables and reflect change in the objective function due to incremental *constraint* relaxation.

5.2. These shadow costs indicate the effect on the objective function of the various *constraints*. The shadow costs related to the system power balance represent the marginal *energy* costs and refer to a location where the market requirement for *energy* is posted, i.e. to the central market place. These shadow costs present an equivalent to System marginal cost in classic unit commitment formulation.

5.3. The Marginal *Energy* Cost for each interval t is determined as shadow cost (λ_{En}^{t}) for *energy* balance *constraints* and it is the uniform price component for all market participants and pricing locations.

5.4. The Marginal Clearing Price is calculated as the As-*Bid* cost of the Marginal Resource.

SECTION 6 NODAL ENERGY DISPATCH PRICES (LOCATIONAL MARGINAL PRICES)

6.1. *Load* and generating unit contributions to the system power balance differ with respect to *network energy* losses and eventual transmission congestion. The *energy* prices are differentiated according to specific conditions of actual power injections and withdrawals at market participant locations. In general, *energy* prices are different at each *network* node, i.e. they present *nodal energy dispatch prices* or Locational Marginal Prices (LMP). In a widely accepted formulation, the *energy* LMP present the marginal cost of serving the incremental *load* at the price location by all available resources of the system.

6.2. The LMP is used to settle the market and is calculated in each run by SCDD. The LMP is calculated for each generator and participating *load*.

[2] NGR is a device that has a continuous operating range from a negative to a positive power injection; i.e., it can operate continuously as either consumes load or provides power, and it can seamlessly switch between generating and consuming electrical energy. NGR functions like a generation resource and can provide energy and AS services.

6.3. To support the settlement, the *energy* LMP is calculated at all pricing locations. Each pricing location corresponds to a single market *network* node where the generator or *load* resources are connected. Pricing locations can also include buses with no resources.

6.4. The Locational Marginal Prices for *energy* are calculated respecting *network* losses and eventual transmission congestion:

$$LMP_{En;node}^{t} = \lambda_{En}^{t} / pf_{node}^{t} + \sum_{line \in N} SF_{line;node} \cdot TSC_{line}^{t} \; ; t \in T; node \in G \cup L$$

6.5. Locational Marginal Prices are the same for *generation* and *load* entities at the same *network* node. The Locational Marginal Prices for *energy* consist of several components:

$LMP_{En;node}^{t} =$	λ_{En}^{t}	- Marginal *Energy* Cost
	$+ \lambda_{En}^{t} \cdot (1 - pf_{node}^{t}) / pf_{node}^{t}$	- Price for marginal *network energy* losses
	$+ \sum_{line \in N} SF_{line;node} \cdot TSC_{line}^{t}$	- Price for marginal *network congestion*.

SECTION 7 RESERVE PRICING

7.1. Similar to *energy* pricing, the marginal cost approach is used for ancillary services pricing. The regional *reserve requirements* are posted as minimum and maximum regional limits. The shadow costs ($\lambda_{ASReg}^{AS;t}$) for posted regional *reserve requirements* present the corresponding shadow costs that are calculated as a by-product of the optimization process. These shadow costs present ancillary service Regional Clearing Prices for each ancillary service. These Regional Clearing Prices are used for *reserve* pricing purposes.

7.2. The regional *reserve shadow price* can be expressed the sum of "Reserve Clearing Price" $(ASMP_{ASReg}^{AS;t})$ and the "Opportunity Cost" $\left(LOC_{ASReg}^{AS;t}\right)$ as:

$$\lambda_{ASReg}^{AS;t} = ASMP_{ASReg}^{AS;t} + LOC_{AS\,\mathrm{Re}\,g}^{AS;t}$$

7.3. The "Reserve Clearing Price" is calculated as part of SCDD solution post-processing, as the *reserve offer* price associated with the marginal block that was cleared in the market. Once obtained, then the "Opportunity Cost" is calculated as the difference between the regional *reserve requirement* constraint *shadow price* and the "Reserve Clearing Price".

Resource Reserve Prices

7.4. The Resource Reserve Marginal Prices are calculated as summation of individual regional prices.

7.5. In general, each generating unit and *load* entity can have a different Reserve Marginal Price.

APPENDIX A.1 - MATHEMATICAL NOTATION

- Offer Costs

$C_{unit}^{En;t}(\cdot)$ is unit *energy generation* cost at time interval *t*
$C_{unit}^{RegRaise;t}(\cdot)$ is unit Regulation Raise cost at time interval *t*
$C_{unit}^{RegLower;t}(\cdot)$ is unit Regulation Lower cost at time interval *t*
$C_{unit}^{FFCS;t}(\cdot)$ is unit Fast Frequency Control Services cost at time interval *t*
$C_{load}^{curt}(\cdot)$ is *load* curtailment cost
$C_{load}^{En;t}(\cdot)$ is *load energy* cost at time interval *t*
$C_{constr}^{vio}(\cdot)$ is *constraint* violation cost at time interval *t*

- Energy Requirements

En_{req}^{t} is market *energy* requirement at time interval *t*. This term refers to total unscheduled (forecasted) *load*
ΔEn_{req}^{t} is change in market *energy* requirement at time interval *t*

- Reserve Requirements

$\underline{Reg}_{ASreq}^{Raise;t}$ is Regulation Raise minimum requirement for *reserve region* at time interval *t*
$\overline{Reg}_{ASreq}^{Lower;t}$ is Regulation Lower maximum requirement for *reserve region* at time interval *t*
$\underline{Reg}_{ASreq}^{Lower;t}$ is Regulation Lower minimum requirement for *reserve region* at time interval *t*
$\underline{Res}_{ASreq}^{t}$ is Frequency Control minimum requirement for *reserve region* at time interval *t*

- Product Quantities

En_{unit}^{t} is unit *energy generation* at time interval *t*
En_{load}^{t} is *load energy* consumption at time interval *t*
$Reg_{unit}^{Raise;t}$ is unit Regulation Raise capacity at time interval *t*
$Reg_{unit}^{Lower;t}$ is unit Regulation Lower capacity at time interval *t*
Res_{unit}^{t} is unit Reserve at time interval *t*
$FFCS_{unit}^{t}$ is unit Fast Frequency Control Services quantity at time interval *t*
CV_{constr}^{t} is *constraint* violation amount at time interval *t*

- Offer/Bid Limits

$\overline{En}_{res}^{t}$	is unit/load maximal *energy generation* at time interval *t*
$\underline{En}_{res}^{t}$	is unit/load minimal *energy generation* at time interval *t*
$\overline{Reg}_{unit}^{Raise;t}$	is unit maximal Regulation Raise capacity at time interval *t*
$\overline{Reg}_{unit}^{Lower;t}$	is unit maximal Regulation Lower capacity at time interval *t*
$\overline{Res}_{unit}^{t}$	is unit maximal contingency *reserve* at time interval *t*

- Resource Capacities

EnH_{unit}^{t}	is unit *energy generation* high limit at time interval *t*
EnL_{unit}^{t}	is unit *energy generation* low limit at time interval *t*
$\overline{En}_{unit}^{T}$	is unit *energy generation* maximum over time horizon *T*
EnH_{load}^{t}	is *load energy* consumption high limit at time interval *t*
EnL_{load}^{t}	is *load energy* consumption low limit at time interval *t*
$RegH_{unit}^{t}$	is unit regulation high limit at time interval *t*
$RegL_{unit}^{t}$	is unit regulation low limit at time interval *t*
$RRL_{unit/load}^{Up}$	is ramp-limited maximum increase of additional energy schedule for reserve
$RRL_{unit/load}^{Dn}$	is ramp-limited maximum decrease of additional energy schedule for reserve

- Ramping Rates

RR_{unit}^{Up}	is unit *energy* Up *ramp rate*
RR_{unit}^{Dn}	is unit *energy* Down *ramp rate*
RR_{unit}^{RegUp}	is unit Regulation Raise *ramp rate*
RR_{unit}^{RegDn}	is unit Regulation Lower *ramp rate*
RR_{unit}^{Res}	is unit Reserve *ramp rate*
RR_{load}^{Up}	is *load energy* Up *ramp rate*
RR_{load}^{Dn}	is *load energy* Down *ramp rate*

- Time Domains

T_{dom}^{En}	is *energy* ramping time domain
T_{dom}^{Reg}	is regulation ramping time domain
T_{dom}^{Res}	is *reserve* ramping Up time
T_{dom}^{En}	is *energy* ramping time domain
T^{AS}	is Ancillary Service ramping time

- Network Loss Model

En_{loss}^{t}	are *network energy* losses at time interval *t*
$En_{loss}^{base;t}$	are base *network energy* losses at time interval *t*
ΔEn_{loss}^{t}	is change of *network energy* losses at time interval *t*
$En_{unit/load}^{base;t}$	is unit/*load* base operating point at time interval *t*
α_{node}^{t}	is loss sensitivity factor for *node* or loss sensitivity to the change of *generation* in the *node* at time interval *t*
$pf_{unit/load}^{t}$	is loss penalty factor for *unit/load* at time interval *t*

- Transmission System Model

P_{line}^{t}	is line actual power flow at time interval *t*
SF_{line}^{node}	is shift factor for transmission *line* and *network node*
$P_{line}^{base;t}$	is line base power flow at time interval *t*
P_{node}^{t}	is actual *generation*/consumption at time interval *t*
$P_{node}^{base,t}$	is *unit*/*load* base *generation*/consumption at time interval *t*
$\underline{P}_{line}^{t}$	is line minimal power flow limit at time interval *t*
$\overline{P}_{line}^{t}$	is line maximal power flow limit at time interval *t*
$line \in N$	is the set of *transmission lines*
$node \in NN$	is the set of *network nodes*

- Commodity Prices

$LMP_{node}^{En;t}$	is Locational Marginal Price for *energy* at *network node* at time interval *t*
$LMP_{Pnode}^{En;t}$	is Locational Marginal Price for *energy* at Pricing Location at time interval *t*
CV_{constr}^{t}	is Constraint Violation for constraint *constr* at time interval *t*

- Market Constituents

$unit \in G$	is the set of online generating units
$load \in L$	is the set of dispatchable *loads*

- Market Timeline

$t \in T$	is scheduling time horizon *T* divided into time intervals *t*
Δ_t	is the duration of the time interval *t*
ρ_t	is the duration of the time interval *t* as a fraction of an hour.

APPENDIX A.2 - COMBINED ENERGY AND RESERVE RAMPING EXAMPLE

To put a numerical example for equations for Resource Combined *Energy* and Reserve Ramping under Section 4.3, let us say that a unit has maximum ramp up rate of 6 MW per minute; that would mean the unit can ramp its power output at maximum 30 MW up from its initial condition (5 minutes times 6 MW per minute equals 30 MW, that is the maximum amount for $RLU_i^{En}(t)$ value).

Let us say that initial condition P(t-1) is 50 MW. In case when there are no *reserve* awards, unit can reach 80 MW at the end of *dispatch interval.*

Now let us assume that the unit for the same interval is awarded 10 MW of Regulation Raise.

In case the unit is called to provide Regulation Raise service (*reserve* award is activated into *energy*), then the unit has to ramp up those ten MWs during the interval, and that ramping comes in addition to ramping of unit's *energy* award.

Model will ensure that *energy* schedule P(t) is no more than the 50 MW (initial condition) + 30 MW (*energy* ramping) – 10 MW (regulation award) for the end of the interval.

So in case where this unit is called for delivery of 10 MW of Regulation Raise award, unit would be able to reach new set point set by *AGC.*

In an example, if a unit is awarded by Market at 65 MW of *energy* for particular interval, and then 10 MW of Regulation Raise, then if the Regulation Raise is activated by the *System Operator*, it would be possible for unit to ramp from its initial condition to the new set point of 65 MW +10 MW = 75 MW, as ramping requirement for the interval of 75 MW – 50 MW (initial condition) = 25 MW, that the unit can ramp in less than five minutes (with max ramping capability of 30 MW over the length of *Dispatch Interval*).

APPENDIX A.3 - SELF-SCHEDULED ENERGY DISPATCH CURTAILMENT

Per current *System Operator* operational practice, in a scenario where a group of *self-scheduled energy generating units* is self-scheduled at multiple points of a multi-leg radial connection, and the curtailment has to be performed for the group, there is specific rule for curtailing the individual units. In such a case, the units are curtailed proportionally to their self-scheduled MW, regardless of economics. To illustrate that practice, example for *self-scheduled energy* is provided below.

In the example shown in Figure 3 below, units D and E have the lowest priority and will be cut first. In this example the Shift Factors for Generators D and E, with respect to the flow on line 2 are assumed to be 1, so the total curtailed MW amount (5 MW) is equal to the MW flow relief (5 MW) of the line 2. This curtailment of 5 MW is distributed among units D and E proportionally to their self-scheduled MWs,, i.e. unit D is getting 40% of the total curtailment, while unit E is getting 60% of the relief.

It is important to note the assumptions for this processing:

- Processing only applies to units that are self-scheduled
- Units subject to this processing are having the same priority
- Units are not subject to ramping *constraints* (being self-scheduled, no ramping *constraints* are applicable per convention)
- Economic impact is to be disregarded (i.e. economical impact of incremental losses or shift factors)
- Minimum operating limit (Pmin) of the units subject to this processing is considered to be zero unless registration data is non-zero. If the *minimum stable load* (Pmin) is greater than zero, then the pro-rata adjustments only applies to Nominated quantities above the Pmin. For example, if the Nomination is 100 MW and the unit Registered Pmin is 40 MW, the amount subject to pro-rata curtailment is 60 MW and that is the coefficient used in the curtailment pro-rata processing (not the Nominated 100 MW).

Generator	Gen Type	Priority #	Segment 1		Segment 2		Segment 3		Segment 4	
			P1	Q1	P2	Q2	P3	Q3	P4	Q4
GEN_A	Scheduled	–	NA	10	-100	20	150	35		
GEN_B	Self-Scheduled	1	NA	15						
GEN_C	Scheduled	–	NA	150	0	265				
GEN_D	Self-Scheduled	2	NA	20						
GEN_E	Self-Scheduled	2	NA	30						

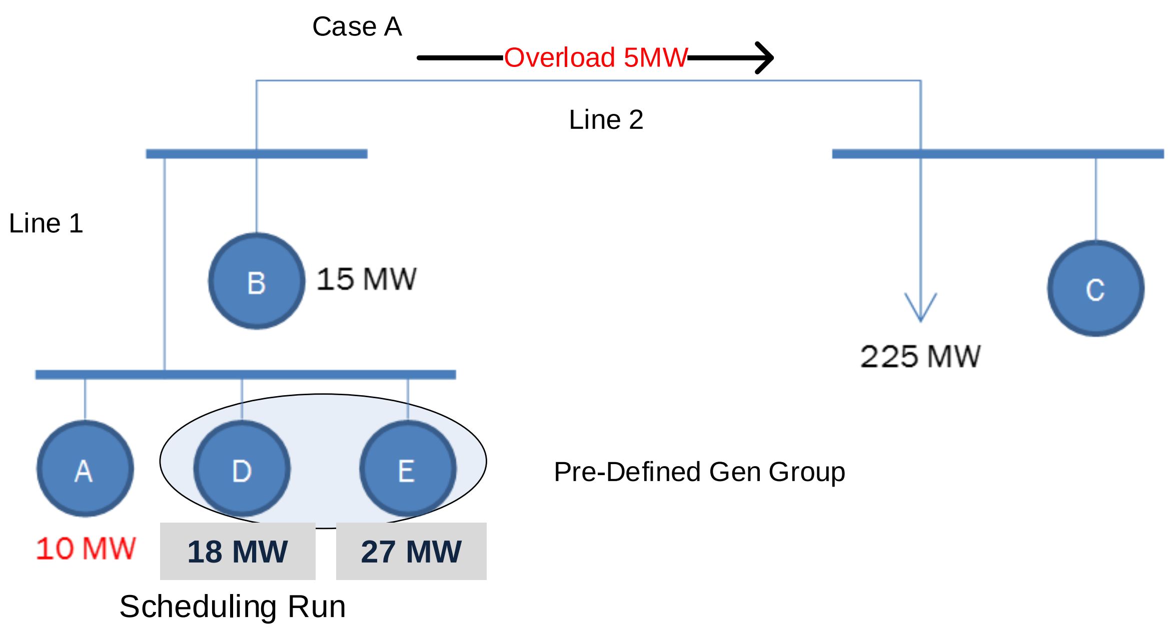

Figure 3: Example of pro-rata curtailment of electrically identical units

In order to provide such functionality, and disregard the economics of the case, the following constraint can be added to the model:

- (En(E,t)-Pmin(E,t)/(Pnom(E,t)- Pmin(E,t)) = (En(D,t)- Pmin(D,t))/(Pnom(D,t)- Pmin(D,t));

Where *En(D,t),Pmin(D,t), Pnom(D,t)* are awarded MW, Pmin and Nominated (self scheduled) MW respectively, for unit *D* in time interval *t*.

While the constraint seems trivial, it will be written only for units that are satisfying the above assumptions, plus the additional assumptions as follows:

- Units have to be defined to belong to a special “pro-rata group”; each unit within the group is subject to prorata processing that links units within the group with the equations above.
- “Pro rata” Groups are defined ahead of time. There can be many groups, and many units within each group, but one unit can belong to only one “pro-rata group”.

Such approach would also satisfy scenario as listed in Figure 4 below:

Generator	Gen Type	Priority #	Segment 1		Segment 2		Segment 3		Segment 4	
			P1	Q1	P2	Q2	P3	Q3	P4	Q4
GEN_A	Scheduled	—	NA	10	-100	20	150	35		
GEN_B	Self-Scheduled	2	NA	15						
GEN_C	Scheduled	—	NA	150	0	265				
GEN_D	Self-Scheduled	1	NA	20						
GEN_E	Self-Scheduled	2	NA	30						

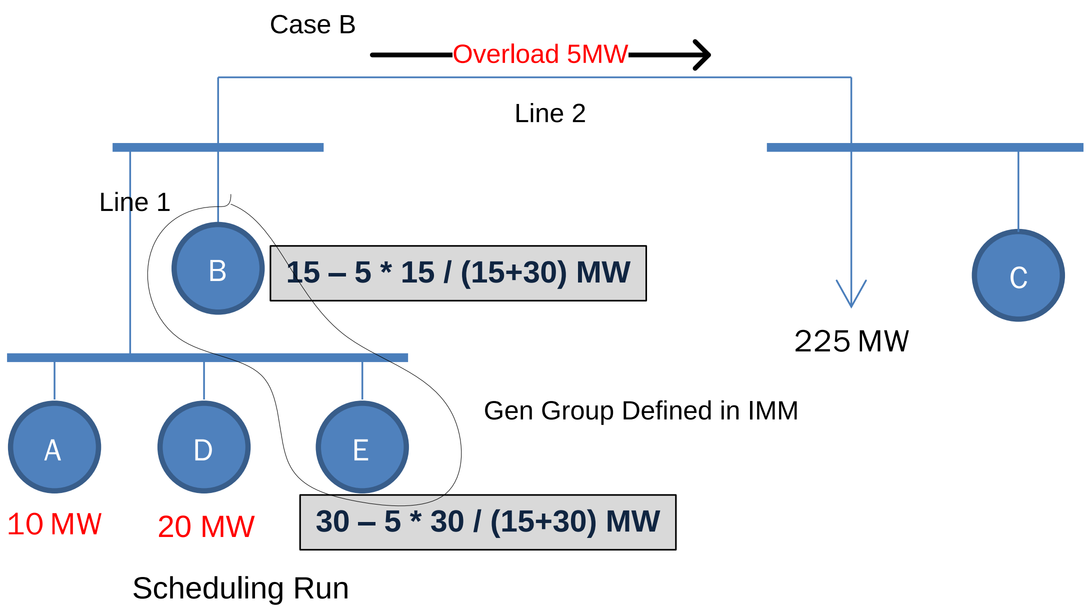

Figure 4: Example of pro-rata curtailment of electrically different units

However, the approach described so far might not be suitable for scenario as listed in Figure 5. For this scenario the pro-rate processing should not be applied (i.e. if the shift factors of the units within the group do not have the same sign).

Generator	Gen Type	Priority #	Segment 1		Segment 2		Segment 3		Segment 4	
			P1	Q1	P2	Q2	P3	Q3	P4	Q4
GEN_A	Scheduled	—	NA	10	-100	20	150	35		
GEN_B	Self-Scheduled	2	NA	15						
GEN_C	Scheduled	—	NA	150	0	265				
GEN_D	Self-Scheduled	1	NA	20						
GEN_E	Self-Scheduled	2	NA	30						

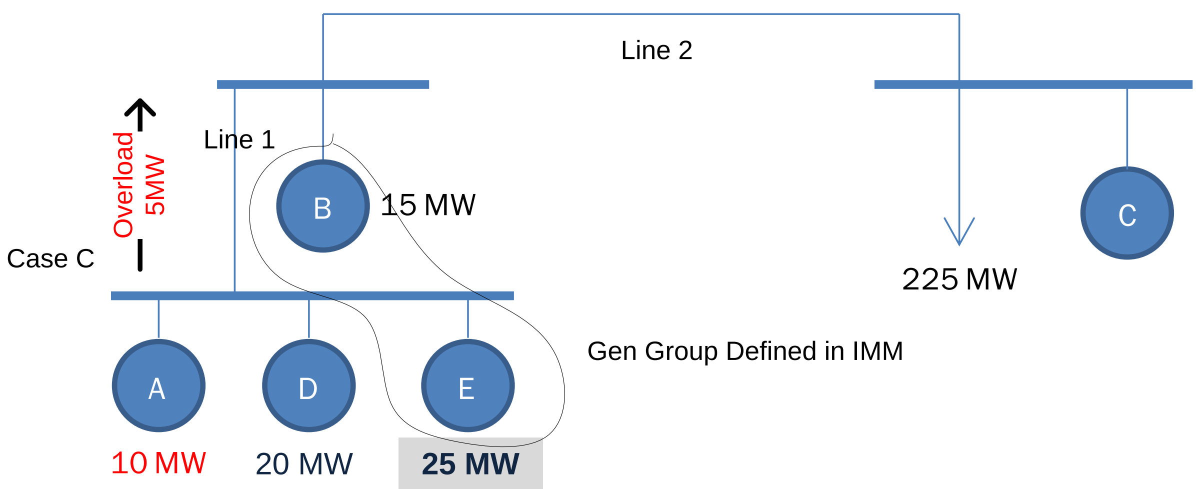

Figure 5: Example of pro-rata curtailment of electrically different units with shift factors of different sign

The approach would also not be suitable for scenario that would involve congestion both on lines 1 and 2 at the same time.

Under such scenarios, where the shift factors of the units within the group are of different sign with respect to a *network constraint,* only the units with same sign shift factors (those providing counter-flow to the congestion) will be subject to pro-rata curtailment.